Domain Walls with Non-Abelian Clouds

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1 Introduction

Soliton Dynamics: Important for **Nonperturbative Effects**

**Brane-World** = Our world on a **Topological Defect** in higher dim. spacetime

**Part of Supersymmetry (SUSY) preserved** → **BPS state**

Similarity to **D-branes** in string theory as BPS states

Parameters of the Solution = **Moduli**

→ **Massless fields** on the world volume

**Moduli dynamics** = **Effective field theory** of massless fields

\[ \begin{align*}
\phi_- & \quad \phi_+ \\
\end{align*} \]

If 2 or more minima of a potential → **Kink** connecting 2 minima can occur

Domain wall position is a moduli (wave function localized around the wall)
Non-Abelian orientational moduli

Single instanton in $SU(N)$ gauge theory

$$A_\mu = U \begin{pmatrix} A^{BPST}_\mu (x_0, \rho) & 0 \\ 0 & 0_{N-2} \end{pmatrix} U^\dagger, \quad U \in \frac{SU(N)}{SU(N-2) \times U(1)}$$

Single vortex moduli in $U(N)$ gauge theory with $N$ Higgs fields

$$H = U \begin{pmatrix} H^{ANO}(z) & 0 \\ 0 & \sqrt{c} 1_{N-1} \end{pmatrix} U^\dagger, \quad F_{12} = U \begin{pmatrix} F^{ANO}_{12}(z) & 0 \\ 0 & 0_{N-1} \end{pmatrix} U^\dagger$$

$$U \in \frac{SU(N)}{SU(N-1) \times U(1)} \simeq CP^{N-1}$$


... 

**Non-Abelian clouds**: non-Abelian moduli (around monopoles, E.Weinberg)

Our purpose:

- Study Domain walls with non-Abelian orientational moduli
- Non-Abelian moduli occur when Higgs masses are (partially) degenerate
- Two cases:
1. $U(1)$ gauge theory with $N_F$ Higgs fields with mass matrix 
   $M = \text{diag}(m_1, 0, \cdots, 0, -m_2)$

2. $U(N)$ gauge theory with $N_F = 2N$ Higgs fields with $N$ masses $-m$
   and $N$ masses $m$ \quad (N = 2 \text{ case: Shifman-Yung, Phys.Rev.} \textbf{D70} (2004) 025013)

Results

1. Domain walls in these models with (partially) degenerate masses for Higgs scalars have normalizable non-Abelian Nambu-Goldstone (NG) modes, which are called Non-Abelian clouds.

2. Effective Lagrangians as nonlinear sigma models are explicitly obtained, with their metric and Kähler potential.

3. When walls coincide, all the massless modes are localized at the walls with identical wave functions.

4. When walls separate, we find non-Abelian clouds which spread between two domain walls.

5. When all the walls coincide in the $U(N)$ gauge model, symmetry breaking $SU(N)_L \times SU(N)_R \times U(1) \rightarrow SU(N)_V$ gives $U(N)_A$
**NG modes.** In addition, there are $N^2 - 1$ **quasi-NG modes** besides 1 NG mode for broken translation. All these modes have identical wave function and are localized at the wall.

6. When $n$ walls separate in the $U(N)$ gauge model, off-diagonal elements of $U(n)$ NG modes have wave function spreading between two separated walls (**non-Abelian clouds**). Some quasi-NG modes turn to NG modes because of further symmetry breaking $U(n)_{V} \rightarrow U(1)^n_{V}$.

7. The **number of massless modes remain unchanged** as wall positions change.

8. In $4 + 1$-dimensions, we **dualize** the effective theory on the $3 + 1$ dimensional world-volume to the supersymmetric Freedman-Townsend model of **2-form fields** valued in $U(N)$.

9. **Moduli matrix** approach is extremely useful to describe non-Abelian clouds of domain walls.

**2 Models, BPS equations and the moduli matrix**

**SUSY $U(N_C)$ Gauge Theory with $N_F$ Higgs fields**
Higgs fields $H$ as an $N_C \times N_F$ matrix, adjoint scalar $\Sigma$

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - V$$

$$\mathcal{L}_{\text{kin}} = \text{Tr} \left( -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{g^2} D_\mu \Sigma D^\mu \Sigma + D^\mu H (D_\mu H)^\dagger \right)$$

$$V = \text{Tr} \left[ \frac{g^2}{4} (c1 - HH^\dagger)^2 + (\Sigma H - HM)(\Sigma H - HM)^\dagger \right]$$

Gauge coupling $g$, Fayet-Iliopoulos parameter $c$, diagonal mass matrix $M$

Bogomol'nyi bound (dependence on $y$ only)

$$E = \int_{-\infty}^{\infty} dy \text{Tr} \left[ (D_y H - HM + \Sigma H)^2 + \frac{1}{g^2} (D_y \Sigma - \frac{g^2}{2} (c1 - HH^\dagger))^2 \right]$$

$$+ c D_y \Sigma \geq c \left[ \text{Tr} \Sigma(\infty) - \text{Tr} \Sigma(-\infty) \right]$$

Lower bound is saturated if the BPS equations are satisfied

$$D_y H = HM - \Sigma H, \quad D_y \Sigma = g^2 (c1 - HH^\dagger) / 2$$

Solution of BPS equations

$$H = S^{-1}(y) H_0 e^{M_y}, \quad \Sigma + i W_y = S^{-1}(y) \partial_y S(y)$$
**Moduli matrix** \( H_0: N_C \times N_F \) constant complex matrix of rank \( N_C \),

contains all the moduli parameters \( \phi^i \)

Remaining BPS eq. (Master eq.):

\[
\Omega \equiv SS^\dagger, \quad \Omega_0 \equiv \frac{1}{c} H_0 H_0^\dagger
\]

\[
\partial_y (\Omega^{-1} \partial_y \Omega) = cg^2 \left( 1_{N_C} - \Omega^{-1} \Omega_0 \right)
\]

Explicit exact solution at \( g^2 \rightarrow \infty \) limit: \( \Omega \rightarrow \Omega_0 \)

Equivalence class defined by \( V \)-transformations (same physical fields)

\[
H_0 \rightarrow V H_0 \quad \text{and} \quad S(y) \rightarrow V S(y) \quad \text{with} \quad V \in GL(N_C, C)
\]

Total moduli space

\[
\{ H_0 | H_0 \sim V H_0, V \in GL(N_C, C) \} \cong \frac{SU(N_F)}{SU(N_C) \times SU(N_F - N_C) \times U(1)}
\]

Isozumi-Nitta-Ohashi-Sakai, Phys.Rev.Lett.93 161601 (2004); ⋯

**Effective Lagrangian**

Kähler potential of the nonlinear sigma model for moduli fields \( \phi^i(x) \)

\[
K(\phi, \phi^*) = \int_{-\infty}^{\infty} dy \left[ K(y, \phi, \phi^*) - K_{ct}(y, \phi) - \tilde{K}_{ct}(y, \phi^*) \right],
\]
$\mathcal{K}(y, \phi, \phi^*) = \text{Tr} \left[ c \log \Omega + c \Omega^{-1} \Omega_0 + \frac{1}{2g^2} (\Omega^{-1} \partial_y \Omega)^2 \right]$

$\mathcal{K}_{ct}, \bar{\mathcal{K}}_{ct}$: counter terms (to eliminate divergence by Kähler transformation)

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**Figure 1:** Continuous series of wall solutions parametrized by $G/H$ are obtained when trajectories pass near the continuously degenerate vacua.

**Domain walls and local structure of the moduli space**

Broken global symmetry $G \to H$: continuously degenerate vacua $G/H$.

If a path of a wall configuration passes near the continuously degenerate vacua, another solution can be obtained by acting the global symmetry $G$. 
→ a **continuous series of solutions** parametrized by $G/H$ non-Abelian Nambu-Goldstone modes of $G/H$ are **localized** around the wall since $G$ fixes the two isolated vacua

3 Non-Abelian Clouds in Abelian Gauge Theories

A simple example: $N_F = 4$ with $M = \text{diag} (m, m\epsilon/2, -m\epsilon/2, -m)$

![Figure 2: Vacua for various cases of mass configurations plotted in the three-dimensional space of Higgs fields $h_i^2, i = 1, 2, 3$ with $\sum_{i=1}^4 h_i^2 = 1$. (a) non-degenerate massive vacua, (b) massive degenerate and non-degenerate vacua, (c) massless vacuum](image-url)
4 isolated vacua $\langle 1 \rangle, \cdots, \langle 4 \rangle$, flavor symmetry $U(1)^3 (\epsilon \neq 0)$

Explicit solution at $g^2 \to \infty$ limit: $H = \frac{1}{\sqrt{\Omega_0}} H_0 e^{My}$

By using V-trnsformations, moduli parameters are $\varphi_1, \varphi_2, \varphi_3$

$$H_0 = (1, e^{\varphi_1}, e^{\varphi_1+\varphi_2}, e^{\varphi_1+\varphi_2+\varphi_3}) = (1, \phi_2, \phi_3, \phi_4)$$

Domain wall trajectories in terms of $|h_i|^2$ of $H = \sqrt{c}(h_1, h_2, h_3, h_4)$ in the target space $\mathbb{C}P^3$ (as Toric diagram)
Wall configuration in terms of $\Sigma$

![Graphs of $\Sigma$ and density of the Kähler metric](image)

(a) $\epsilon = 1/3$  
(b) $\epsilon = 1/20$  
(c) $\epsilon = 1/100$

**Figure 3:** Configuration of $\Sigma$ (first row) and density of the Kähler metric of $\varphi_1$, $\varphi_2$ and $\varphi_3$ (second row). Moduli parameters are $(\varphi_1, \varphi_2, \varphi_3) = (20, 0, -20)$ and $m = 1$.

**Well-separated walls:** $\varphi_i$ is the **i-th wall position** and **phase difference**

wave function is localized around the **i-th wall**

As $\epsilon$ decreases, $\varphi_2$ wave function spreads between **2 walls**

$\rightarrow$ **Non-Abelian clouds**
$\epsilon \to 0$ (degenerate mass) limit: enhanced flavor symmetry $U(1)^2 \times SU(2)$

2 isolated vacua $\langle 1 \rangle$, $\langle 4 \rangle$ and a degenerate vacuum $\langle 2 - 3 \rangle$

(with vacuum moduli $CP^1 \simeq SU(2)/U(1)$)

When domain walls are coincident,

all 6 massless modes are localized at the wall with identical wave functions

When domain walls are separated, 6 massless modes consist of

positions of two walls: $\{|\phi_2|^2 + |\phi_3|^2, |\phi_4|^2\}$ (1 NG, 1 qNG)

Localized at each wall

N-G bosons for $U_1(1) \times U_2(1) \times SU(2)/U(1)$ (4 NG)

$U_1(1), U_2(1)$: localized at each wall

$SU(2)/U(1)$: spread between 2 walls

Effective action of non-Abelian clouds

$U(1)$ gauge theory and $N_F$ flavors with masses

$M = (m_1, 0, 0, \cdots, 0, -m_2)$, $m_1, m_2 > 0$

Full moduli space: $\mathcal{M} \simeq \mathbb{C}^* \times \mathbb{C}^{N_F-2}$
Choosing the center of mass at $y = 0$, the moduli matrix is

$$H_0 = (1, \phi_2, \phi_3, \cdots, \phi_{N_F-1}, 1)$$

Positions of 2 walls

$$y_1 = \frac{1}{2m_1} \log |\phi|^2, \quad y_2 = -\frac{1}{2m_2} \log |\phi|^2$$

Distance between 2 walls: $R$

$$R = y_1 - y_2 = \frac{1}{\mu} \log |\phi|^2, \quad \mu \equiv \frac{2m_1 m_2}{m_1 + m_2}$$

Effective Lagrangian for non-Abelian orientational moduli (Non-Abelian clouds)

$$\phi = e^{(\mu R + i\xi)/2} n, \quad |n|^2 = 1$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} f''(\mu R) \left[ \mu^2 (\partial_\mu R)^2 + (\partial_\mu \xi - 2in^\dagger \partial_\mu n)^2 \right]$$

$$+ f'(\mu R) \left[ |\partial_\mu n|^2 - |n^\dagger \partial_\mu n|^2 \right]$$

$$f(\mu R) = \begin{cases} 
\frac{c\mu R^2}{2} - (d_1 + d_2)c\mu R + \mathcal{O}(1) & \text{for } R \to \infty \\
A e^{\mu R} + \mathcal{O}(e^{2\mu R}) & \text{for } R \to -\infty 
\end{cases}$$
4 The Generalized Shifman-Yung Model

\textbf{U}(N)\ gauge theory and \( N_F = 2N \) Higgs fields with mass matrix

\[ M = \frac{1}{2} \text{diag}(m, \cdots, m, -m, \cdots, -m) \]

Non-Abelian flavor symmetry: \( SU(N)_L \times SU(N)_R \times U(1)_A \)

\((k, N - k)\) vacuum \((0 \leq k \leq N)\):

\[ \Sigma|_{\text{vacuum}} = \frac{1}{2} \text{diag}(m, \cdots, m, -m, \cdots, -m), \]

\[ H|_{\text{vacuum}} = \sqrt{c} \begin{pmatrix} 1_k & 0 & 0_k & 0 \\ 0 & 0_{N-k} & 0 & 1_{N-k} \end{pmatrix} \]

Flavor symmetry breaking: \( 4k(N - k) \) Nambu-Goldstone modes

\( SU(N)_L \to SU(k)_{C+L} \times SU(N - k)_L \times U(1)_{C+L} \), \( SU(N)_R \to SU(k)_R \times SU(N - k)_{C+R} \times U(1)_{C+R} \)

Maximal number of walls: \((N, 0) \to (0, N)\) contains \( N \) walls

\textbf{Moduli matrix} can be written with \( e^\phi \in GL(N, C) \)

\[ H_0 = \sqrt{c}(1_N, e^\phi) \sim \sqrt{c}(e^{-\phi}, 1_N) \]
Full moduli space: \( \mathcal{M} \simeq GL(N, C) \cong U(N)^C \simeq C^* \times SL(N, C) \)

\( 2N^2 \) real parameters

**Flavor symmetry**: \( G = SU(N)_L \times SU(N)_R \times U(1)_A \)

\[ e^\phi \rightarrow e^{i\alpha} g_L e^\phi g_R^\dagger, \quad g_L \in SU(N)_L, \ g_R \in SU(N)_R, \ e^{i\alpha} \in U(1)_A \]

**Nambu-Goldstone (NG) modes and quasi-NG modes**

Flavor symmetry \( G \) can bring the moduli matrix diagonal

\[ e^\phi \rightarrow \text{diag.}(v_1, v_2, \cdots, v_N) = m \text{ diag.}(y_1, y_2, \cdots, y_N) \]

\( y_i \): position of \( i \)-th wall

When all \( y_i \) coincide, diagonal subgroup \( SU(N)_V \) is maintained

broken global symmetry: \( G \rightarrow SU(N)_V \), translation

NG bosons: \( N^2 + 1 \)

qNG bosons: \( N^2 - 1 \)

wave functions are localized at the wall and are identical


When some \( y_i \)'s separate, part of the non-Abelian group \( SU(N)_V \) is broken
part of qNG bosons turns into NG bosons,
total number of massless modes unchanged \(2N^2\)

When all \(y_i\) are different (completely separated walls)
broken global symmetry: \(G \rightarrow U(1)^{N-1}_V\), translation
NG bosons: \(2N^2 - (N - 1)\),
qNG bosons: \(N - 1\) (relative positions \(y_i\) without overall translation)

**total number of massless modes unchanged** \(2N^2\)

Wave functions of qNG, translation, and \(U(1)^N_A\): localized at each wall
Wave functions of other NG: spread between walls

Different from D-branes, (in disagreement with Shifman-Yung)

**Effective action of domain walls**

Kähler potential is only a function of \(\hat{x} = \frac{1}{2} \log(e^\phi e^{\phi^\dagger})\)

\[
K(\phi, \phi^\dagger) = \text{Tr}[F(\hat{x})]
\]

\(F\) is independent of the size \(N\) of matrix \(\rightarrow N = 1\) case

Nambu-Goldstone modes for broken translation and \(U(1)\) phase
\[ K(\phi, \phi^\dagger) = \frac{c}{m} \text{Tr}[\hat{x}^2] = \frac{c}{4m} \text{Tr} \left[ (\log(e^\phi e^{\phi^\dagger}))^2 \right] \]

Localization properties in strong coupling limit

A parametrization of Moduli matrix with \( U_L \in U(N), U_R \in U(N) \)

\[ e^\phi = U_L e^{\phi_0} U_R^\dagger, \quad \phi_0 = m \text{ diag } (y_1, y_2, \cdots, y_N) \]

Strong coupling limit \( g^2 \to \infty : \Omega = \Omega_0 \)

For well-separated walls, Density of the Kähler metric is

\[
\mathcal{K}_{ij^\ast} \partial_\mu \phi^i \partial^\mu \phi^{j^\ast} = \frac{c}{4} \sum_{r} \frac{N}{\cosh^2(m(y - y_r))} \left| (\tau_{\mu})_{rr} \right|^2 + c \sum_{r \neq s} \frac{\cosh^2 \left( \frac{m}{2} (y_r - y_s) \right) \left| (\tau_{\mu})_{rs} \right|^2}{\cosh(m(y - y_r)) \cosh(m(y - y_s))}
\]

\( (\tau_{\mu})_{rs} \approx \left\{ \begin{array}{ll}
-(U_R^\dagger \partial_\mu U_R)_{rs} & \text{for } r > s, \\
m \partial_\mu y_r + (U_L^\dagger \partial_\mu U_L)_{rr} - (U_R^\dagger \partial_\mu U_R)_{rr} & \text{for } r = s, \\
(U_L^\dagger \partial_\mu U_L)_{rs} & \text{for } r < s.
\end{array} \right. \)
Diagonal mode $(\tau_\mu)_{rr}$: localized at $r$-th wall $y = y_r$

Off-diagonal mode $(\tau_\mu)_{rs}$: spread between $y = y_r$ and $y = y_s$

→ Non-Abelian clouds

5 Duality and Two-Form Fields

(Abelian) duality on the $(d - 1) + 1$ dimensional world-volume of walls

Moduli = massless spin 0 field $\phi \leftrightarrow d - 2$ form dual field $B_{d-2}$

$d = 3$ case: $N_C = 2$ with $M = \text{diag.}(m, m, -m, -m)/2$

4 NG bosons are dualized to give gauge fields at the linearized level


separated wall $\rightarrow$ wave function spreading between walls

massless NG scalars, no massive gauge fields

$4 + 1$ spacetime (Maximal dimensions): Realistic for brane-world

$4$ SUSY manifest in $3 + 1$ world-volume:

Chiral superfield $\Phi(x, \theta, \bar{\theta}) \leftrightarrow$ chiral spinor superfield $B_\alpha(x, \theta, \bar{\theta})$

$$B^\alpha(y, \theta) = \psi^\alpha(y) + \frac{1}{2} \theta^\alpha (C(y) + i D(y)) + \frac{1}{2} (\sigma^{\mu\nu})^{\alpha\beta} \theta_\beta B_{\mu\nu}(y) + \theta \theta \eta^\alpha(y)$$
\[ \tilde{B}_\alpha(y^\dagger, \bar{\theta}) = \bar{\psi}_\alpha(y^\dagger) + \frac{1}{2} \bar{\theta}_\alpha(C(y^\dagger) - i D(y^\dagger)) + \frac{1}{2} (\sigma^{\mu\nu})_{\dot{\alpha}\dot{\beta}} \bar{\theta}^\dot{\beta} B_{\mu\nu}(y^\dagger) + \bar{\theta} \bar{\theta} \bar{\eta}_\alpha(y^\dagger) \]

\[(\sigma^{\mu\nu}) = \frac{1}{4} (\sigma^{\mu\nu} - \sigma^{\nu\mu}) \]

\[ y^\mu \equiv x^\mu + i \theta \sigma^\mu \bar{\theta}, \quad y^{\mu\dagger} = x^\mu - i \theta \sigma^\mu \bar{\theta} \]


2-form field valued in \( G = U(N) \):

\[ B_\alpha(x, \theta, \bar{\theta}) = B^A_\alpha(x, \theta, \bar{\theta}) T_A \]

\( U(N) \)-valued real auxiliary vector superfield

\[ A(x, \theta, \bar{\theta}) = A^A(x, \theta, \bar{\theta}) T_A \]

(First-order) Lagrangian for \( B_\alpha \)

\[ \mathcal{L} = -\frac{1}{2f} \left[ \int d^2 \theta \ Tr(W^\alpha B_\alpha) + \int d^2 \bar{\theta} \ Tr(\bar{W}_\dot{\alpha} \bar{B}^\dot{\alpha}) \right] + \frac{1}{4f} \int d^4 \theta \ Tr A^2 \]

\[ W_\alpha = -\frac{1}{4} \bar{D} \bar{D} (e^{-A} D_\alpha e^A), \quad \bar{W}_\dot{\alpha} = \frac{1}{4} D D (e^A \bar{D}_\dot{\alpha} e^{-A}) \]

Invariant under (Abelian) tensor gauge transformation with \( \Omega(x, \theta, \bar{\theta}) \)

\[ \delta B_\alpha = -\frac{i}{4} \bar{D} \bar{D} D_\alpha (e^{-A} \Omega), \quad \delta \bar{B}^\dot{\alpha} = -\frac{i}{4} D D \bar{D}^\dot{\alpha} (\Omega e^{-A}), \quad \delta A = 0 \]

covariant spinor derivative

\[ D_\alpha = D_\alpha + [e^{-A} D_\alpha e^A, \cdot] \]
Invariant under \textbf{global} $U(N)$ transformation with $g \in U(N)$

$$
B_\alpha \rightarrow B'_\alpha = g^{-1} B_\alpha g, \quad \bar{B}_\dot{\alpha} \rightarrow \bar{B}'_{\dot{\alpha}} = g^{-1} \bar{B}_{\dot{\alpha}} g
$$

$$
A \rightarrow A' = g^{-1} A g, \quad W_\alpha \rightarrow W'_\alpha = g^{-1} W_\alpha g
$$

Eliminating $A \rightarrow$ second-order Lagrangian for $B_\alpha$

Eliminating $B_\alpha \rightarrow GL(N, C)$ nonlinear sigma model

$$
-4W_\alpha(x, \theta, \bar{\theta}) = 0 \rightarrow e^{A(x,\theta,\bar{\theta})} = e^{\phi(x,\theta,\bar{\theta})} e^{\phi^\dagger(x,\theta,\bar{\theta})}, \quad \bar{D}_{\dot{\alpha}} \phi(x, \theta, \bar{\theta}) = 0
$$

$$
\mathcal{L} = \int d^4 \theta \frac{1}{4f} \text{Tr} \left[(\log(e^\phi e^{\phi^\dagger}))^2\right]
$$

$$
1/f = c/m, \quad B^A_{\mu\nu}: \text{NG bosons of } U(N), \quad C^A: \text{quasi-NG bosons}
$$
6 Conclusion

1. Domain walls in these models with (partially) degenerate masses for Higgs scalars have normalizable non-Abelian Nambu-Goldstone (NG) modes, which are called Non-Abelian clouds.

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